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Robust Multiuser Detection for DS-CDMA Systems in Bandlimited White Laplace Noise

Vinay Kumar Pamula[1] and Anil Kumar Tipparti[2]

[1]Department of ECE, UCEK, JNTUK, Kakinada, India
[2]Department of ECE, CMRIT, Hyderabad, India

***Abstract*—The direct sequence-code division multiple accesses (DS-CDMA) signals are transmitted frequently over mobile communication channels that introduce heavy-tailed bandlimited white Laplace noise. Combined effect of multiple access interference (MAI), inter-symbol interference (ISI) and Laplace noisedeteriorates the system performance. This paper presents the performance of proposed *M*-estimator based detector by evaluating probability of error. Simulation results show that the proposed *M*-decorrelator performs better, in the presence of heavy-tailed Laplace noise, when compared to least squares, Huber and Hampel *M*-estimator based detectors.**

***Index Terms*— CDMA, impulsive noise, Laplace noise, *M*-estimator, multiuser detection, bit error rate.**

I. Introduction

Mobile communications research has explored the potential benefits of multiuser detection (MUD) for code division multiple access (CDMA) communication systems [1]. The optimal multiuser detectors have led to the developments of the various linear multiuser detectors with the assumption of Gaussian noisethough various experimental results have confirmed that many realistic channels are impulsive in nature [2], [3].The problem of robust MUD in non-Gaussian channels has been addressed in the literature [4], [5], [6], which were developed based on the Huber, Hampel, and a new *M*-estimator based detectors, respectively, by modeling the impulsive noise with two-term Gaussian mixture model [4]. The Laplace noise model can also be commonly used as a model for impulsive noise which prevalent in both indoor and outdoor radio and undersea transmission scenarios [7], [8]. The performance analysis of binary data detection systems in Laplace noise is also important and that leads to performance analysis of ultra wideband (UWB) communication systems operating in multiuser interference [7], [8]. As far as author's knowledge is concerned, little attention has been paid to the studies on MUD techniques in Laplace noise. Hence, this paper considers multiple access interference (MAI) mitigation in direct sequence-CDMA (DS-CDMA) channels with non-Gaussian ambient noise modeled by wideband white Laplace noise.

The remaining part of the paper is organized as follows: DS-CDMA system innon-Gaussian impulsive noise environment is presented in Section II. Section III presents the penalty function and influence function of the proposed *M*-estimator, and the asymptotic variance of the minimax decorrelating detector with proposed estimator. Section IV presents the simulation results to study the performance of proposed *M*-decorrelator in heavy-tailed impulsive noise modeled by Laplace distribution. Finally, conclusions are drawn in Section V.

II. System Model

An L-user DS-CDMA system operating with coherent binary phase shift keying (BPSK) modulation scheme is considered in this paper. If all the L-users are active, the received baseband, continuous-time signal is a superposition of all L signals given by [6]

$$r(t) = X(t) + z(t) \tag{1}$$

where $X(t)$ is the useful signal comprised of the data signals of L active users and the $z(t)$ is the additive white Gaussian noise (AWGN). The signal $X(t)$ can be written as

$$X(t) = \sum_{l=1}^{L} w_l(i) \sum_{i=0}^{M-1} b_l(i) s_l(t - iT_s - \tau_l) \tag{2}$$

where M is the number of data symbols per user in the data frame of interest, $T_s$is the symbol interval, $b_l(i)$ is the i^{th} bit of the l^{th} user, $w_l(i)$ is the received signal amplitude, $s_l(t)$ is the normalized signaling waveform of the l^{th} user and τ_l is the delay of the l^{th} user . It is assumed that the signaling interval of each user is T_s seconds, and the input alphabet is antipodal binary: [-1, +1]. Further, direct-sequence spread spectrum (DSSS) multiple access (MA) signatures can be written as

$$s_l(t) = \sum_{n=1}^{N} a_n^l u_{T_c}(t-(n-1)T_c), \quad t \in [0, T_s] \tag{3}$$

where $\{a_n^l\}_{n=1}^{N}$ is a signature sequence of +1s and -1s assigned to the l^{th} user, and $u_{T_c}(t)$ is a unit-amplitude pulse of duration T_c with $T_s = NT_c$. At the receiver, the received signal $r(t)$ is first filtered by a chip-matched filter and then sampled at the chip rate, $1/T_c$. The resulting discrete-time signal sample corresponding to the n^{th} chip of the i^{th} symbol is given by [3]

$$r_n(i) = \sqrt{\frac{2}{T_c}} \int_{iT_s+nT_c}^{iT_s+(n+1)T_c} r(t) e^{-j\omega_c t} dt, n = 1...N . \tag{4}$$

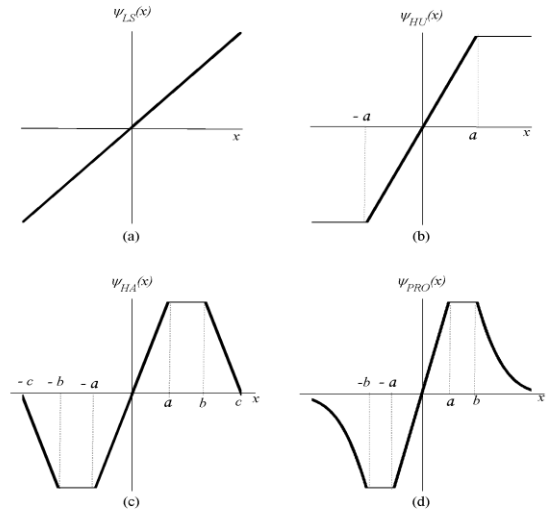

Fig. 1 Influence functions of (a) the linear decorrelating detector, (b) Huber estimator, (c) Hampel estimator, and (d) the proposed estimator.

For synchronous case (i.e., $\tau_1 = \tau_2 = ... = \tau_l = 0$), assuming that the fading process for each user varies at a slower rate that the magnitude and phase can taken to be constant over the duration of a bit, (4) can be expressed in matrix notation as [3]

$$\underline{\boldsymbol{r}}(i) = \underline{A}\underline{\boldsymbol{\theta}}(i) + \underline{\boldsymbol{z}}(i) \tag{5}$$

where

$$\underline{\boldsymbol{r}}(i) = \left[\boldsymbol{r}_1(i),...,\boldsymbol{r}_N(i)\right]^T \tag{6}$$

$$\underline{\boldsymbol{z}}(i) = \left[\boldsymbol{z}_1(i),...,\boldsymbol{z}_N(i)\right]^T \tag{7}$$

$$\underline{\boldsymbol{\theta}}(i) = \frac{1}{\sqrt{N}}\left[\boldsymbol{b}_1(i)\boldsymbol{g}_1(i),...,\boldsymbol{b}_L(i)\boldsymbol{g}_L(i)\right]^T \tag{8}$$

with $\boldsymbol{z}_n(i)$ is a sequence of independent and identically distributed (i.i.d.) complex random variables whose in-phase and quadrature components are independent non-Gaussian random variables with probability density function (PDF) of this noise model has the form

$$f(x;\Lambda,\lambda) = \frac{1}{2\Lambda}\exp\left(-\frac{|x-\lambda|}{\Lambda}\right) \tag{9}$$

where λ is the location parameter and Λ is the shape parameter and is related to the variance of the variable as $\Lambda^2 = \sigma^2/2$.

III. *M*-DECORRELATOR

In this section, the penalty function and influence function of the proposed [6]*M*-estimator are presented.

A. M-estimator

An *M*-estimator is, a generalization of usual maximum likelihood estimates, used to estimate the unknown parameters $\theta_1, \theta_2, ...\theta_L$ (where $\theta = \underline{A}\underline{b}$) by minimizing a sum of function $\rho(\cdot)$ of the residuals [4]

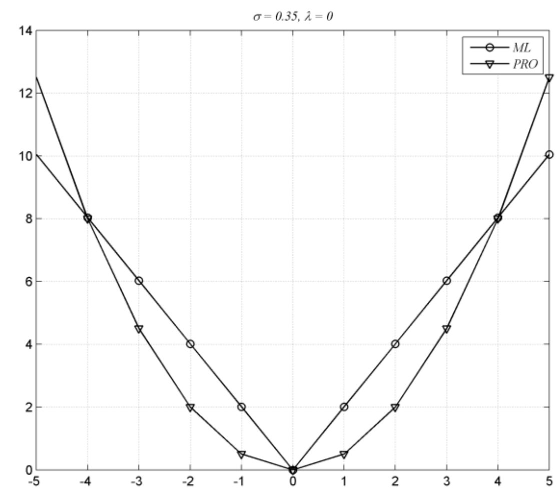

Fig. 2 Penalty function of the *ML* estimator of the Laplace noise model, and penalty function of the proposed *M*-estimator for σ = 0.35 and λ = 0.

$$\hat{\boldsymbol{\theta}} = \underset{\boldsymbol{\theta}\in\Re^L}{\arg\min} \sum_{j=1}^{N} \rho\left(r_j - \sum_{l=1}^{L} s_j^l \theta_l\right) \tag{10}$$

where ρ is a symmetric, positive-definite function with a unique minimum at zero, and is chosen to be less increasing than square and N is the processing gain. An influence function $\psi(\cdot) = \rho'(\cdot)$ is proposed (as shown in Fig. 1 (d) along with other influence functions), such that it yields a solution that is not sensitive to outlying measurements. The penalty function and the corresponding influence function of the proposed*M*-estimator are given by [6]

$$\rho_{MH}(x) = \begin{cases} \dfrac{x^2}{2} & \text{for } |x| \le a \\ \dfrac{a^2}{2} - a|x| & \text{for } a < |x| \le b \\ d - \dfrac{ab}{2}\exp\left(1 - \dfrac{|x|^2}{b^2}\right) & \text{for } |x| > b \end{cases} \tag{11}$$

$$\psi_{MH}(x) = \begin{cases} x & \text{for } |x| \le a \\ a\,\mathrm{sgn}(x) & \text{for } a < |x| \le b \\ \dfrac{a}{b} x \exp\left(1 - \dfrac{x^2}{b^2}\right) & \text{for } |x| > b \end{cases} \tag{12}$$

where the choice of the constants *a* and *b* depends on the robustness measures.

B. Asymptotic probability of error

Theasymptotic probability of error for the class of decorrelating detectors, for large processing gain *N*, is given by [4]

$$P_e^l \equiv \Pr\left(\hat{\theta}_l < 0 \middle| \theta_l > 0\right) = Q\left(\frac{A_l}{\upsilon\sqrt{\left[\mathbf{R}^{-1}\right]_{ll}}}\right) \tag{13}$$

where $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-t^2/2}dt$ is the Gaussian Q-function [9],

$$\upsilon^2 = \frac{\int_{-\infty}^{\infty} \psi^2(u) f(u) du}{\left[\int_{-\infty}^{\infty} \psi'(u) f(u) du\right]^2} = \frac{\upsilon_n^2}{\upsilon_d^2} \tag{14}$$

and $\mathbf{R} = \mathbf{S}^T\mathbf{S}$ with $\mathbf{S}$ is an $N \times L$ matrix of columns a_l.The asymptotic variance is obtained by substituting (15) and (22) into (14). With assumption that the errors followed a known long-tailed distribution with density $f(\cdot)$, we would be led to using the corresponding maximum likelihood (ML) estimator, because it is asymptotically efficient. In fact, the ML estimator is an M-estimator with penalty function, $\rho_{ML}(x) = -\log f(x)$. For small measurement x, both $\rho_{ML}(x)$ and $\rho_{PRO}(x)$ coincide with $\rho_{LS}(x)$. For $|x| \to \infty$, we have $\rho_{MH}(x) \to d$. Therefore, for input samples with large amplitude, the penalty function of the proposed detector does not increase. However, the penalty function of the ML estimator increases with $|x|$, and therefore, the ML estimator is not robust [6]. Fig. 2 shows the penalty function of ML estimator for Laplace noise and also the penalty function of the proposed detector with $\sigma = 0.35$ and $\lambda = 0$.

For the proposed minimax decorrelating detector, the numerator and denominator integrals of (13) can be computed as

$$\upsilon_n^2 = n_1 + n_2 + n_3 + n_4 + n_5 + n_6 \tag{15}$$

where

$$nMH_1 = \frac{a^2}{2b^2c}\left[\begin{array}{l}\gamma\left(\frac{b^2+8bc+16c^2}{8c^2},1\right)\frac{b^4\exp\left(\frac{b^2}{8c^2}-\frac{\lambda}{c}+2\right)}{8c} \\ +\gamma\left(\frac{b^2+8bc+16c^2}{8c^2},1/2\right)\frac{b^5\exp\left(\frac{b^2}{8c^2}-\frac{\lambda}{c}+2\right)}{2^{11/2}c^2} \\ +\gamma\left(\frac{b^2+8bc+16c^2}{8c^2},3/2\right)\frac{b^3\exp\left(\frac{b^2}{8c^2}-\frac{\lambda}{c}+2\right)}{2^{5/2}}\end{array}\right] \tag{16}$$

$$nMH_2 = \frac{a^2}{2}\left[\exp\left(\frac{-a-\lambda}{c}\right) - \exp\left(\frac{-b-\lambda}{c}\right)\right] \tag{17}$$

$$nMH_3 = c^2\exp\left(\frac{-\lambda}{c}\right) - \frac{1}{2}\exp\left(\frac{-a-\lambda}{c}\right)\left[a^2 + 2ac + 2c^2\right] \tag{18}$$

$$nMH_4 = \frac{-a^2}{2}\left[\exp\left(\frac{-a+\lambda}{c}\right) - \exp\left(\frac{\lambda}{c}\right)\right] \tag{19}$$

$$nMH_5 = \frac{-a^2}{2}\left[\exp\left(\frac{-a+\lambda}{c}\right) - \exp\left(\frac{-b+\lambda}{c}\right)\right] \tag{20}$$

$$nMH_6 = \frac{a^2}{2b^2c}\left[\begin{array}{l}\gamma\left(\frac{b^2+8bc+16c^2}{8c^2},1\right)\frac{b^4\exp\left(\frac{b^2}{8c^2}+\frac{\lambda}{c}+2\right)}{8c} \\ +\gamma\left(\frac{b^2+8bc+16c^2}{8c^2},1/2\right)\frac{b^5\exp\left(\frac{b^2}{8c^2}+\frac{\lambda}{c}+2\right)}{2^{11/2}c^2} \\ +\gamma\left(\frac{b^2+8bc+16c^2}{8c^2},3/2\right)\frac{b^3\exp\left(\frac{b^2}{8c^2}+\frac{\lambda}{c}+2\right)}{2^{5/2}}\end{array}\right] \tag{21}$$

and

$$\upsilon_d^2 = dMH_1 + dMH_2 + dMH_3 + dMH_4 \tag{22}$$

where

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$$dMH_1 = \frac{1}{2c}\begin{bmatrix} -\frac{a}{2}\sqrt{\pi}\exp\left(\frac{b^2}{4c^2}-\frac{\lambda}{c}+1\right)erf\left(\frac{b}{2c}+1\right) \\ -\gamma\left(\frac{b^2+4bc+4c^2}{4c^2},\frac{1}{2}\right)\exp\left(\frac{b^2}{4c^2}-\frac{\lambda}{c}+1\right)\left(\frac{a}{2c}\right) \\ +\gamma\left(\frac{b^2+4bc+4c^2}{4c^2},1\right)\exp\left(\frac{b^2}{4c^2}-\frac{\lambda}{c}+1\right)\left(\frac{a}{b}\right) \\ +\frac{a}{2}a\sqrt{\pi}\exp\left(\frac{b^2}{4c^2}-\frac{\lambda}{c}+1\right) \end{bmatrix} \quad (23)$$

$$dMH_2 = \frac{1}{2}\left[\exp\left(-\frac{\lambda}{c}\right)-\exp\left(\frac{-a-\lambda}{c}\right)\right] \quad (24)$$

$$dMH_3 = \frac{1}{2}\left[\exp\left(\frac{\lambda}{c}\right)-\exp\left(\frac{-a+\lambda}{c}\right)\right] \quad (25)$$

$$dMH_4 = \frac{1}{2c}\begin{bmatrix} -\frac{a}{2}\sqrt{\pi}\exp\left(\frac{b^2}{4c^2}-\frac{\lambda}{c}+1\right)erf\left(\frac{b}{2c}+1\right) \\ +\gamma\left(\frac{b^2+4bc+4c^2}{4c^2},\frac{1}{2}\right)\exp\left(\frac{b^2}{4c^2}-\frac{\lambda}{c}+1\right)\left(\frac{a}{2c}\right) \\ -\gamma\left(\frac{b^2+4bc+4c^2}{4c^2},1\right)\exp\left(\frac{b^2}{4c^2}-\frac{\lambda}{c}+1\right)\left(\frac{a}{b}\right) \\ +\frac{a}{2}a\sqrt{\pi}\exp\left(\frac{b^2}{4c^2}-\frac{\lambda}{c}+1\right) \end{bmatrix} \quad (26)$$

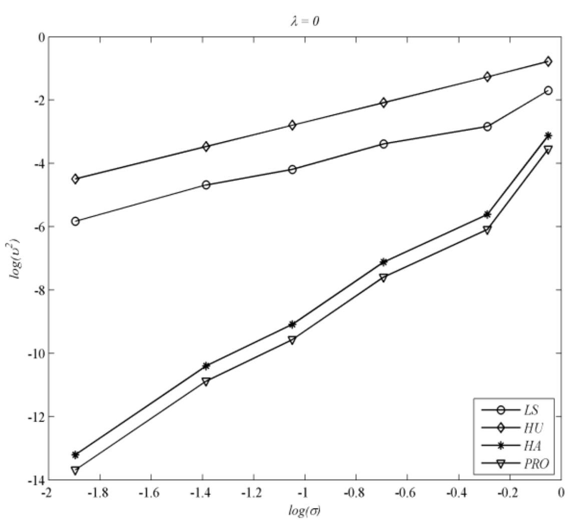

Fig. 3 Asymptotic variance υ^2 of the four decorrelating detectors as a function of σ with $\lambda = 0$.

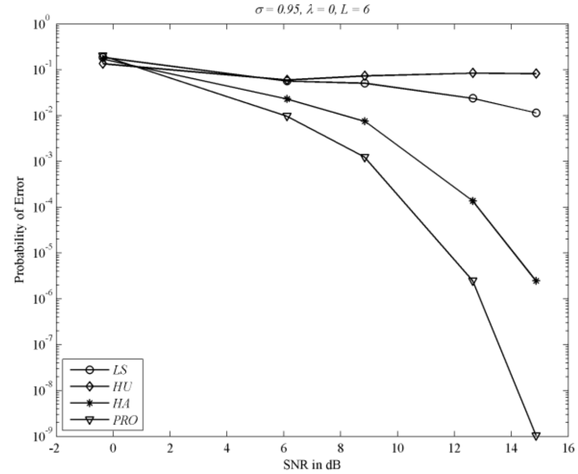

Fig. 4 Average probability of error versus SNR for user # 1 for linear decorrelating detector (*LS*), minimax detector with Huber (*HU*), Hampel (HA) and proposed (*PRO*)*M*-estimator in synchronous DS-CDMA channel with Laplace noise (σ = 0.95, λ = 0), *N* =31.

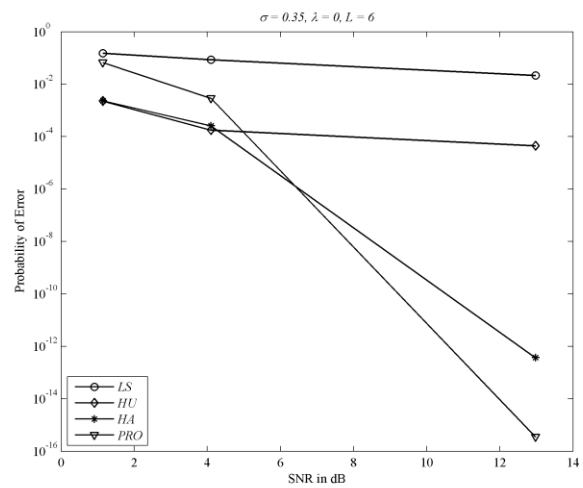

Fig. 5 Average probability of error versus SNR for user # 1 for linear decorrelating detector (*LS*), minimax detector with Huber (*HU*), Hampel (HA) and proposed (*PRO*)*M*-estimator in synchronous DS-CDMA channel with Laplace noise (σ = 0.35, λ = 0), *N* =31.

Here, $\gamma(,\cdot,)$ is the upper incomplete gamma function [9].

In Fig. 3, the asymptotic variance of the minimax decorrelating detector with the proposed (*PRO*) *M*-estimator in comparison with the asymptotic variance of the linear decorrelating detector (*LS*), Huber (*HU*) and Hampel (*HA*) estimator based detectors. From Fig. 3, it is clear that the minimax decorrelating detector with the proposed influence function outperforms other decorrelating detectors.

IV. SIMULATION RESULTS

In this section, the performance of *M*-decorrelator is presented by computing the probability of error, (13), for different values of Laplace noise parameters.

Performance of *M*-decorrelator with different influence functions is shown in Fig. 4 and Fig. 5.

In Fig. 4, the probability of error versus the signal-to-noise ratio (SNR) corresponding to the user 1 under perfect power control of a synchronous DS-CDMA system with six users (L = 6) and a processing gain, N = 31 is plotted for Laplace noise (σ = 0.95, λ = 0). In Fig. 3, the average probability of error is plotted for moderate impulsive noise (ε = 0.01). From Fig. 3, it is clear that when the noise variance is high (σ = 0.95), the performance of the proposed decorrelator is better compared to other decorrelating detectors.

Similarly, in Fig. 5, the average probability of error is plotted for (σ = 0.35, λ = 0). When the noise variance is reduced to 0.35, the performance of the proposed detector is superior compared to least squares detector and slightly inferior compared to minimax detector with Huber and Hampel estimator based detectors at lower values of SNR less than 6 dB. However, the proposed detector outperforms with increased SNR.

Simulation results show that the proposed *M*-estimator based detector performs well compared to linear multiuser detector, minimax detector with Huber and Hampel estimator based detectors in Laplace noise.

V. Concluding Remarks

A multiuser detection technique for DS-CDMA systems inadditive bandlimited white Laplace noise environment is presented. An *M*-estimator based multiuser detector is proposed and its performance is analyzed by computing probability of error. Simulation results show that the proposed multiuser detector offers significant performance gain over the linear multiuser detector and the minimax decorrelating detector with Huber and Hampel *M*-estimators, in heavy-tailed Laplace noise.

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