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Controllability Problems and Stability Problems of Nonlinear Discrete Dynamical Systems

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Abstract - Current mathematical control theory emphasizes controllability as a key concept. A dynamical system's controllability, in general, refers to the ability to guide it from any first state to any final state by a set of acceptable instructions. There are several definitions of controllability in the literature, all of which are very dependent on the kind of dynamical system under consideration.

Keywords: Stability, Nonlinear analysis.

1 INTRODUCTION

Controllable and observable core ideas, as well as weaker concepts like stabilizability and detectability, are crucial to solving a number of key optimal control issues. This research examines the restricted-controllability1-D and 2-D control systems specified issue for nonlinear finite-dimensional discrete-time in confined domains.

Up to this point, controllability in linear dynamical systems with incessant and separate time has been fully examined in various research (for a list of articles, see (Klamka, 1991b; 1993; 1995)). When dealing with nonlinear dynamical systems, especially those that include restricted controllers, this isn't necessarily the However, there are just a few articles that address the issue of controlled controllability for incessant or blinking nonlinear or linear dynamic.

1-d and 2-d systems For nonlinear limited-dimensional discrete with steady parameters(Graves, 1950; Robinson, 1986), this led to the identification of local controllability problems It is possible to establish and test appropriate criteria for restricted controllability in confined domains using nonlinear functional analysis mapping theorems or linear estimate approaches using nonlinear functional

1.1 Dynamic Systems, Discreteness and Nonlinearity

One of the most important characteristics of dynamical systems is the concept of time. Thereby "time" differs from the current concept of physical time, which is subject to numerous restrictions due contemporary scientific theory and experimentation. Instead, we are interested in Newton's classical characterization of time, which is as a generic mathematical parameter. When the system changes "states," this time parameter is required to monitor the transitions. What makes a dynamical system unique is its reliance on time. In the subsequently part, we will define the terms ‘state' and ‘state-space,' which serve as a foundation for understanding dynamic systems.

Definition 1. *When a dynamical system's state-space is a set of time-dependent variables that take values in a vector space V, it is enough to know the values of the variables at any given time t_0. The values of these variables determine the current state of the dynamical system.*

The “state-space” framework for describing dynamical systems has its origins in classical mechanics philosophy. Its archetypal expression may be found in Pierre Simon de Laplace's statement:

“There would be no uncertainty for it if it had an intelligence that could comprehend all of nature's forces, as well as the conditions of the creatures that make it up.

2. REVIEW OF LITERATURE AND CONTROLLABILITY PROBLEMS AND STABILITY PROBLEMS OF NONLINEAR DISCRETE DYNAMICAL SYSTEMS

Jerzy Klamka et. al. [1] presented Discrete nonlinear finite-dimensional discrete 1D and 2D control systems with constant coefficients are studied in order to answer issues about local restricted controllability. The mapping theorems of nonlinear functional analysis and linear approximation methods are used to create and establish these required conditions for restricted controllability Controllability requirements for unconstrained discrete systems with restricted controls are therefore expanded to cover both 1-D and 2-D discrete systems with restricted controls.

Arash Hassibi et. al. [2] in this paper events" that occur asynchronously drive dynamical systems. Even though the duration of the time period T is infinite, the event rates are regarded to be limited or at Through technical advances in digital and communication systems, they are becoming more and more important to the control sector. These include asynchronous control systems, distributed control systems, and parallelized numerical methods. a queuing system, and a Researchers at the University of California, Los Angeles have developed an advanced Lyapunov-based theory for dynamical systems that may be controlled by solving problems of bilinear matrix inequality (BMI) or linear matrix inequality (LMI). The method's efficacy is demonstrated through examples.

Jerzy Klamka et. al. [3] revealed in this paper Control systems with constant coefficients are examined in this article. They have a linear, continuous-time, finite- A three-part structure is used throughout the essay. For starters, there are some basic definitions of stability, as well as essential and sufficient Second, controllability of dynamical control systems is described, and necessary and sufficient criteria for controllability are provided, utilising a controllability matrix as a reference. There is also a discussion of the crucial situation of controllability with restricted controls Observables are the focus of the third portion. With the observability matrix, required and sufficient observability criteria are defined here. In conclusion, a few observations are made about special instances of linear control system stability, controllability, and observability. However, it is important to highlight that all of the results are presented without evidence, but with appropriate literature

Eurika Kaiser et. al. [4] proposed The Koopman and Perron Frobenius transport operators are profoundly altering how we think about dynamics because of their capacity to provide linear representations for even very nonlinear dynamics. Working with them numerically, on the other hand, is challenging due to the fact that transport operators are always changing. As a result, dynamic mode decomposition has quickly become a common numerical technique for estimating the Koopman operator. Increasing data quantities and DMD's adaptability to linear algebra have led to the rapid development of similar methods in recent years. There is an overview of key advancements in data-driven characterization of transportation operators in this chapter. In addition, new approaches in big data and machine learning are presented, including recent advances in sparsity and control.

James Kapinski et. al. [5] indicated to show nonlinear and hybrid dynamical systems' constancy and to get presentation constraints on system behaviours, Lyapunov functions are utilised, however finding Lyapunov functions in general is a difficult process. For nonlinear and hybrid systems, we present a investigate-based advance for finding Lyapunov functions. It is indeed possible to find candidates for the Lyapunov function by constructing linear programming (LP) optimization problems based on actual executions (such as those gained from simulation). Improvement of candidate Lyapunov functions by employing a global optimizer that is driven by Lie derivatives of the candidate Lyapunov function. An SMT solver is used to refine the analysis using counterexamples. As soon as there are no counterexamples, an arithmetic solver is used to verify the soundness of the analysis A wide range of nonlinear dynamical systems may be investigated with this, including hybrid systems, polynomial dynamical systems, and even transcendental dynamics. This method has been shown to be effective in two vehicle powertrain control scenarios.

Samia Charfeddine et. al. [6] indicated When employing output feedback linearization and sliding mode control architecture, this study analyses the control of nonlinear disturbed polynomial systems. The objective is to achieve asymptotic stability of a volatile equilibrium point As far as we know, the Byrnes–Isidori normal form for class is identical to that for class.

Conventional feedback linearizing control techniques may fail in the face of model errors and/or process dynamic disturbances. Sliding mode and meta-heuristic approaches are utilised to create a robust control system. Difficulties in the system output are minimised as a result of the asymptotically stable dynamic behaviour of this control architecture. As a benchmark, a continuous stirred tank reactor was used to conduct numerical simulation analysis to evaluate the efficacy and efficiency of the proposed approach (CSTR).

X. Koutsoukos et. al. [7] hierarchical control structure for piecewise linear hybrid systems is provided. This is achieved by utilising piecewise linear maps to connect the continuous and discrete components of the dynamic system and linear difference equations for the acicular. There are techniques for synpaper of dynamical controllers, as well as formulations for the control design issue as a regulator. Finite automata are used to model the control requirements. Consideration is given to both static and dynamic requirements. A finite automaton and a linear programming method are used to implement the Illustrations for the technique are based on simulations of a tank system

Thiagop.Chagas et. al. [8] presented researchers are studying how chaotic sets in nonlinear discrete-time dynamical systems stabilise periodic orbits. As a consequence, a novel control rule has been developed that approximates the issue to that of stabilising linear time-periodic systems using contemporary control theory. It was evaluated to a best Delayed Feedback Control when it was analysed using numerical simulations, proving its merits both theoretically and practically.

Ya Tian et. al. [9] revealed in this paper in this paper, A class of continuous-time dynamical systems' global asymptotic behaviour is examined. It is possible to investigate chaotic control and chaotic synchronisation by obtaining not only the final bounds on system solutions, but we can also acquire the rate of trajectories travelling as of the perimeter of a ensnare set to the inside of a ensnare set.

Vipin Kumar et. al. [10] proposed in this paper, on time scales, we show the continuation and uniqueness of the answer to a nonlinear fractional discrepancy equation with nonlinear integral boundary conditions to arrive to these conclusions, As a result of our work, we were able to use the fixed point theories of Banach, Schaefer, Leray Schauder, and Krasnoselskii, among others. Ulam-type Hyer's stability finding is also examined. As a conclusion, we provide two instances to demonstrate the usefulness of our findings and conclusions.

Guilherme Franca et. al. [11] indicated As a result of Nesterov's invention, the gradient descent acceleration approach is extensively employed in machine learning, although its principles are this knowledge gap has been closed recently by the use of a continuous-time dynamical systems perspective in conjunction with gradient techniques for smooth and unconstrained issues. The alternating-direction technique of multipliers is extended to nonsmooth and linearly constrained circumstances by developing nonsmooth dynamical systems connected to variants of the relaxed and accelerated alternating-direction method (ADMM). Two unique versions of the ADMM method are shown here. One is based on Nesterov's acceleration, while the other is built around the heavy ball approach of Polyak. A nonsmooth Lyapunov analysis provides rate-of-convergence findings in convex and substantially convex scenarios for these dynamical systems, illustrating an interesting trade-off between Nesterov and heavy ball acceleration.

Parikha Mehrotra et. al. [12] presented Although it is been around for a while, the stability problem intended for a certain class of linear incessant time Switched linear systems with nonstop and discrete descriptions have also been included in this study' So, a common quadratic and nonquadratic Lyapunov function is constructed to solve the arbitrary switching issue in such circumstances. The complexity of studying switched systems with delays is exacerbated by the fact that dynamical systems have invertible temporal delays owing to internal or external causes. The present work, highlights the state of art review of switched systems and underlying methodologies to ensure stabilization of such system with individual systems having stable or unstable dynamics. Further, current status and future directions of possible scope and open challenges in this area are also highlighted for completeness.

Alberto Carrassi et. al. [13] revealed In addition to meteorology and oceanography, these systems are fast expanding to other geosciences and continuum physics fields, according to this article. Solutions for such a system are unusual due to its nonlinear stability over time and their convergence with the system's real chaotic evolution is required. Using a linearized Lorenz system, we show the essential concepts of our method. Dynamic meteorology's Lyapunov exponents are used to analyse two nonlinear prediction-assimilation systems' stabilities. Data-induced stabilisation is critical to the functioning of a system of this type. An observational network, which may be also permanent or data-adaptive, or incorporation techniques that can be either fixed or data-adaptive have a role in determining this to some.

Alexander P. Buslaev et. al. [14] proposed Last decade and early this decade, researchers studied discrete models of movement on one contour, which is comparable to CA184 according to Wolfram cellular automata categorization (WCA). If you look at comparable formulations of issues using contour networks, you have seen that nodes so far, these issues have been understudied. They are, nevertheless, extremely significant in terms of scientific and research applications related to flow theory (traffic, growth of new resources, power metabolism in the body).In this study, the simplest contour network is studied. In this network, there is a common node between the two outlines there are two contours that meet at a same place in the system's supporter (node). Particles travel along their contour according to Wolfram cellular automata rule 184 or 240. Each contour has a shared cell or a point shared by two cells (alternating node). We created techniques for two-contour system research that may be used to the study of contour networks in order to analyse contour networks with more sophisticated architecture. When the system reaches a condition of unrestricted mobility, the following requirement is met (self-organization). According to rule CA184, the self-organization criteria consists of an inequality of □1 + □2 ≤ 1⁄2 while rule CA240 requires an inequality of □1 + □2 ≤ 1 Overall, the existence or absence of self-organization is determined by the system's starting condition, which When particles travel according to distinct laws of motion, they move at varying speeds You also learn about the possibilities for further research and prospective applications.

Venkatesan Govindaraj et. al. [15] indicated in this paper, In this paper, nonlinear fractional dynamical systems with Lipchitzian and non-Lipchitzian nonlinearities are controlled We can assess the nonlinear system's controllability with the use of nonlinear analysis approaches such as fixed point theorems and monotone. In order to demonstrate the findings, examples are supplied As a result, a new type of controllability result has been discovered for fractional dynamical systems with.

Kumpati s. Narendra et. al. [16] presented it shows that neural networks may be used to identify and regulate nonlinear dynamical systems. On the one hand, it focuses on identification models, On the other hand, both static and dynamic back-propagation strategies for parameter adjustment. There is a real need to investigate multilayer and recurrent networks as a whole in the models that are introduced. Simultaneous identification and adaptive control techniques were found to be effective based on simulation. Throughout the article, basic ideas and definitions are given, as well as theoretical problems that must be answered.

R. Bouyekhfa et. al. [17] revealed in this paper According to the study's objectives, it is possible to analyse the asymptotic stability of equilibrium states in nonlinear discrete-time dynamical systems that are time-invariant and nonlinear. Solution based on G-Function idea presented in this article. Derivatives include the definition of new asymptotic stability criteria for such systems and the identification of the asymptotic stability domain for x = 0. The results are illustrated using examples.

Hong Zhang et. al. [18] proposed the dynamics of hybrid dynamical systems subjected to time-dependent perturbations within a short time period are investigated in this work. The null solution's finite-time contractive stability is determined by monotonicity arguments. Last but not least, a numerical simulation is used to show the theoretical analysis.

Zhan-Dong Mei et. al. [19] researchers are going to look at dynamic boundary systems that have feedback on their boundaries in this article A number of regularity requirements are used to demonstrate the well-posedness of the systems under Derivatives of spectral characteristics have also been uncovered Solution of well-posedness and asymptotic behaviour of populace dynamics with limitless birth procedure '$B(t) = \int_0^\infty \beta(a)u(t-\tau, a)da, t \geq 0$'is used in this application. This type of population dynamical system was identified as a semigroup theory research issue that is currently unsolved.

Yonglu Shu et. al. [20] indicated currently, In engineering, chaotic systems and applications based on chaos are often employed. The chaotic control and synchronisation is one of the key structures of these applications. As a result of this research, a unique hyperchaotic There are global exponentially attractive sets, and positively invariant sets as a result of the Lyapunov Theorem with differentials and integrals. It's also possible to estimate the speed of the trajectories It's also possible to estimate the speed of the trajectories There is a theoretical basis for researching chaos management, chaotic synchronisation, and chaos synchronisation for this system by using global exponential attractive sets created in As computer simulations have shown, the proposed method is successful.

2.1 Why to Study Discrete Dynamical System?

There are instances when the independent variable is discrete or it is mathematically advantageous to describe it as such. Discrete-time systems have several uses. A discrete variable represents generation in genetics.

Whether you are studying economics, keep an eye on the price variations from one year to the next, or even one every scenario has a discrete time frame. The variable that indicates age group is a discrete variable if you are interested in Population Dynamics. Discrete-time systems come in two flavours.

Several systems, also including digital computers, digitally analyzers, and financial and stock systems, operate on a discrete time basis. As a result of this, only distinct periods in time are important, and everything else is Examples of discrete-time systems are savings account transactions and the repayment process for bank loans.

2.1.1. Saving Bank Account

In this case, let x(n) be a savings bank balance, and r be the monthly interest rate. Assume that x(n) is equal to the total amount of deposits and withdrawals for the n The equation x(n) fulfils a linear difference equation where the interest is computed monthly based on the beginning balance, for example: n = 0, 1, 2,...

$$x(n+1) = (1+r)x(n) + u(n), x(0) = x_0 \qquad (2.1.1)$$

In this case, x_0 is the starting balance. A linear discrete-time system with linear temporal invariance is described by this equation

2.1.2. Amortization

Paying back a debt by making periodic payments, each of which includes interest and principal reduction, is known as amortisation

P(n) is the unpaid principal after n^{th} payments g(ri). Suppose, for the sake of argument, that interest costs are compounded at a rate of r per payment

In this instance, the outstanding principal p(n + 1) after the (n + 1)st payment equals the outstanding principal (n) after the nth payment, plus the interest rate rp(n) collected throughout the (n + 1)st period, less the final payment gn).

Hence

$$p(n+1) = (1+r)p(n) - g(n), p(0) = Po \qquad (2.1.2)$$

2.2 State Space Description of Discrete-Time System

Simultaneous difference equations (SDE) are useful for describing many systems

$$x(t+1) = f\big(t, x(t)\big), t \in N_0 \triangleq \{0,1,2, \ldots.\} \qquad (2.2.1)$$

Where: $N_0 \times \Omega \to R^n, \Omega \subset R^n$, At the equilibrium point x*, the function is continuously differentiable

As a result, it has a specific

$$x(t+1) = f(x(t)), t \in N_0 \qquad (2.2.1)$$

2.2.1 is a nonlinear autonomous difference equations system, whereas 2.2.2 is a nonlinear non-autonomous dissimilarity equations system. 2.2.2 is a nonlinear autonomous There are several methods for linearizing nonlinear systems, and we will explore them in (see Elaydi [8], page no. 197).

Remark 2.2.1.
If f (t,x) = x* for every t t0, then x* is termed an equilibrium point (2.2.1) in R^n. Assumed to represent the origin 0 in most literature, x* is called the zero solution.*

2.3 Linearization of Nonlinear Systems

Consider system (2.3.1). Let us write $= (f1, f2, \ldots, fn)^T$. The Jacobian matrix of f is defined as

$$\frac{\partial f(t,x)}{\partial x}|x=0$$

$$= \frac{\partial f(t,0)}{\partial x}\begin{pmatrix} \frac{\partial f_1(t,0)}{\partial x_1} & \frac{\partial f_1(t,0)}{\partial x_2} & \cdots & \frac{\partial f_1(t,0)}{\partial x_n} \\ \frac{\partial f_2(t,0)}{\partial x_1} & \frac{\partial f_2(t,0)}{\partial x_2} & \cdots & \frac{\partial f_2(t,0)}{\partial x_n} \\ \vdots & \vdots & & \vdots \\ \frac{\partial f_n(t,0)}{\partial x_1} & \frac{\partial f_n(t,0)}{\partial x_2} & \cdots & \frac{\partial f_n(t,0)}{\partial x_n} \end{pmatrix}$$

For simplicity $\frac{\partial f(t,0)}{\partial x}$ is denoted by $f(t,0)$ Let

$$\frac{\partial f(t,0)}{\partial x} = A(t)$$

And

$$g(t,x) = f(t,x) - A(t)x(t)$$

A variation of this might be worded as follows:

$$x(t+1) = A(t)x(t) + g\big(t, x(t)\big) \qquad (2.3.1)$$

having its linear component

$$x(t+1) = A(t)x(t) \qquad (2.3.2)$$

Where $\big(A(t)\big)t \in N_0$ is a sequences of real $n \times n$ matrices, $(x(t))t \in N_0$ is a sequences of state vectors in $R^n, g\big(t, x(t)\big): N_0 \times \Omega \to R^n, \Omega \subset R^n$ takes the form of a nonlinear function that reflects the perturbation resulting from noise, measurement error, or other external Here it is assumed that $g\big(t,x(t)\big) = 0(x) as \parallel x \parallel \to 0.$
i.e. if given $\in > 0$, there is $\delta > 0$such that

$$\parallel g(t,x) \parallel \leq \epsilon \parallel x \parallel \text{ when ever } \parallel x \parallel < \delta \; and t \in N_0$$

According to equation (2.3.1), the autonomous system may be expressed as follows:

$$x(t+1) = Ax(t) + g\big(x(t)\big) \qquad (2.3.3)$$

Having its linear component

$$x(t+1) = Ax(t) \qquad (2.3.4)$$

where $A = f'(0)$ is the Jacobian matrix of f at 0 and $g(x) = f(x) - Ax$.. Since f is differentiable at 0, we can write $g(x) = 0(x) as \parallel x \parallel \to 0.$. That is,

$$\lim_{\parallel x \parallel \to 0} \frac{\parallel g(x) \parallel}{\parallel x \parallel} = 0$$

2.4 Existence and Uniqueness of Solutions

2.4.1 Solutions of Linear Systems

Due to $x(t_0) = x_0$ linear system (2.4.2) becomes an initial value issue, and the following theorem guarantees that it will have a unique

Theorem 2.4.1.
This is based on Elaydi [8]). A unique solution exists for each$x_0 \in R^n$ and $t_0 \in N_0$ for which (2.4.2) is true. The answer may be written as: $x(t_0) = x_0$

$$x(t) = \left[\prod_{i=t_0}^{t-1} A(i)\right] x_0 \qquad (2.4.1)$$

Where,

$$\prod_{i=t_0}^{t-1} A(i) = A(t-1)A(t-2)\ldots A(t_0), if\ t > t_0$$
$$= I \qquad , if\ t = t_0$$

This is also true for $x(t_0) = x_0$ in the linear autonomous system (2.4.4)

$$x(t) = A^{t-t_0} x_0 \qquad (2.4.2)$$

When it comes to the theory of linear systems, understanding the concept of a basic matrix important

2.4.2 Fundamental Matrices

A matrix with columns that represent solutions to an equation is called Let $\Phi(t)$ (2.4.2). to put it differently...

$$x_1(t), x_2(t), \ldots \ldots, x_n(t)t$$

are solutions of (2.4.2), we write

$$\Phi(t) = [x_1(t)\ x_2(t) \ldots \ldots .. x_n(t)]$$

2.5 DISCRETE VOLTERRA SYSTEMS

In the majority of cases, discrete-time Volterra equations result from the simulation of a true phenomena or from the application of numerical techniques to a Volterra Formula for calculating the Volterra's difference

$$x(t+1) = A(t)x(t) + \sum_{r=0}^{t} B(t,r)x(r), x(0) = x_0 \quad (2.5.1)$$

The Volterra Integrodifferential Equation can be seen as a discrete analogue

$$x(t) = A(t)x(t) + \int_0^t B(t,r)x(r)dr \quad (2.5.2)$$

Regard as the disconcerted equation of (2.5.1)

$$x(t+1) = A(t)x(t) + \sum_{r=0}^{t} B(t,r)x(r) + f(t), x(0) = x_0 \quad (2.5.3)$$

The functions $x(t) \in R^n, A(t), B(t,r)$ and $N_0 \times N_0$ B(t,r) are N_0 matrix functions. T's vector function (f(t)) on N0 is an n- Equation 2.4.1: Define R(t, m) matrix as the unique solution of matrices equation.

$$R(t+1,m) = A(t)R(t,m) + \sum_{r=0}^{t} B(t,r)R(t,m)t \geq m \quad (2.5.4)$$

with R(m, m) = 1 for 0 < m < t.

Elaydi [7] shown that equation (2.5.3) has a unique solution x(t) that may be stated.

$$x(t) = x(t,0,x_0) = R(t,0)x_0 + \sum_{r=0}^{t-1} R(t,r+1)f(r) \quad (2.5.5)$$

2.6 Stability Analysis

Typically, we are interested in methods and criteria that characterise the structure and behaviour of differential system solutions without actually creating or estimating solutions. Due to the fact that many difficulties that arise in practise are nonlinear and intractable, this research is essential. There is also the possibility of using a given, the study of ultimate behaviour of its solutions is one of the pioneer issues (i.e. asymptotic behaviour of discrete systems).

In the first place, let us just review some ideas related to the stability of (see Agarawal [1], Blaydi [8]).

An example is a system of differential equations

$$x(t+1) = f(t, x(t)), x(t_0) = x_0, \quad t \in N_0 \triangleq \{0,1,2,\ldots\} \quad (2.6.1)$$

2.6.1 Stability of Linear Systems

Regard as the linear non-antonomous (time-variant) system known by

$$x(t + 1) = A(t)x(t) \quad (2.6.2)$$

and linear autonomous system (time-invariant) known by

$$x(t + 1) = Ax(t) \quad (2.6.3)$$

These well-known conclusions (Agarwal [1], Elaydi [8]) on the stability of linear systems offer the essential and enough criteria in words of the basic matrices of the systems in question.

2.6.2 (SP) Matrices

The (sp) matrix was recently proposed by Xue and Guo and its utility in the investigation of asymptotic stability of the null solution of linear systems has been demonstrated

$$x(t + 1) = Ax(t)$$

Definition 2.6.2. ((sp) Matrix) We call

$$A \in s = \{A = (a_{ij})_{n\times n}: a_{ij} \geq 0, \sum_{j=1}^{n} a_{ij} \leq 1, \forall i = 1,2,\ldots,n\}$$

a (sp) matrix if there exists$m \in N$ ={1,2,...} and a sequence of subscript sets $\{I_1^{(k)}\}, \{I_2^{(k)}\}, k = 0,1,\ldots, m$ from I = {1,2, ...,n} such that

$$I = I_1^{(0)} \cup I_2^{(0)}, I_1^{(0)} = \left\{i: \sum_{j=1}^{n} a_{ij} < 1\right\}, I_2^{(0)} = \left\{i: \sum_{j=1}^{n} a_{ij} = 1\right\},$$

$$I_1^{(k)} = \left\{i \in I_2^{(k-1)}: \exists j \in I_1^{(k-1)} such\ as\ a_{ij} \neq 0\right\}, I_2^{(k)} = \left\{i \in I_2^{(k-1)}: \forall j \in I_1^{(k-1)} such\ as\ a_{ij} = 0\right\},$$

k= 1,2,.....m-1,

$$I_1^{(m)} = I_2^{(m-1)}, I_2^{(m)} = \emptyset,$$

where $I_1^{(k)}$ and $I_2^{(k)}$, k — 0,1,2, ...,m — 1 are nonempty or $I_2^{(0)} = \emptyset$

2.6.3 Generalized Sub-Radius

It was Czornik [5] who proposed the notions of generalised spectral sub-radius and joint spectral sub-radius, and the relationship between generalized spectral radii and discontinuous time-varying stability A Σ non-empty collection of all real n x n matrixes is denoted by the notation "

For$m \geq 1, \Sigma^m$, contains all the matrices that have m- Σ dimensional products in.

$$\Sigma^m = \{A_1, A_2 \ldots \ldots A_m: A_i \in \Sigma, i = 1,2,\ldots, m\}$$

Denote by $\rho(A)$ the spectral radius and by $\|A\|$a matrix norm of the matrix A. Let $A \in \Sigma^m$. .

2.6.4 Dichotomy

A linear difference equation's dichotomous behaviour can be used to examine an asymptotic link between the solutions of a linear difference equation and a nonlinearly disturbed equation. Difference equations are described as asymptotically comparable if each solution of one system corresponds to a solution of the other.

Take a look at the nonlinear perturbe

$$y(t+1) = A(t)y(t) + g\big(t, y(t)\big), t \in N_0 \qquad (2.6.6)$$

as well as the corresponding collected system

$$x(t + 1) = A(t)x(t) \qquad (2.6.7)$$

2.7 Controllability Analysis

In mathematical control theory, controllability is a key concept to understand Dynamic control methods produce these qualitative features. For time-invariant and time-varying linear control systems, Kalman developed a theory of controllability in the early 1960s ([19]).

According to the definition of a controllable system, it is one that can be directed from any starting point to any ending point by employing a set of permitted controls According to the kind of dynamical control system, the concept of controllability in the literature differs widely. Assume you have a collection of linear difference equations of the following types

$$x(t+1) = Ax(t) + Bu(t), x(0) = x_0, t \in N_0 \qquad (2.7.1)$$

For example, in $(x(t))t \in N_0$ and $(u(t))t \in N_0$ the state vector sequence in R^n and the control vector sequence in R^m are respectively represented by A and B. Here are several definitions of controllability first.

2.7.1 Various Notions and Basic Results of Controllability

Definition 2.7.1. (Complete Controllability)

For any N 0, and any $\in N_0$, starting state, $x(t_0) = x_0$,, if the system (2.7.1) is fully or simply controllable, then there exists a finite time and control (u(t)) such that the end state (xf) is identical to the starting state (x0), then the system (2.7.1) is completely or simply controllable (N).

Definition 2.7.2. (Controllability to Origin)

It is possible to direct a system $t_0 \in N_0$ and $x_0 \in R^n$, (2.7.1) back to the origin if there is a limited time $N > t_0$ and a manage u(t), $t_0 < t < N$ such that x(N) = 0..

Definition 2.7.3. (Local Controllability)

According to the definition of local controllability, there must exist a neighbourhood of the origin such that for each pair of inputs $x_0, x_1 \in \Omega$ there exists an input series $u = (u(0), u(1), \ldots., u(N-1))$, which steers the system from point A to point B x_0 to x_1.

3. CONCLUSION

Various discrete time linear and nonlinear systems' controllability and stability were investigated. By way of illustration, fixed point theorems, inverse functions, and implicit functions may all be used to determine controllability. Aside from the controllability results, we also attempted to develop a computational approach for computing actual steering controls.

- Semi-linear discrete-time system steering control

$$x(t+1) = A(t)x(t) + B(t)u(t) + f\big(t, x(t)\big), t \in N_0 \triangleq \{0,1,2, \ldots\}$$

According to Banach's fixed point theorem, it is well-defined if its linear counterpart can be controlled and its nonlinear function is Lipschitz. A similar analysis shows that controllability and reachability are equal for a system under the same conditions (1). The semi-linear steering control algorithm (1) is provided.

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