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On Solving Transportation Problem Based on Interval Valued Pythagorean Octagonal Neutrosophic Fuzzy Number

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Abstract: The Neutrosophic Set is an extension of the fuzzy set and the Intuitionistic Fuzzy Set which can be deal with imperfect, inconsistent, and indeterminate data in all the related problems. This article proposed the approach has considered a transportation problem based on Interval Valued Pythagorean Octagonal Neutrosophic Fuzzy Number with the numerical illustration.
Keyboard: Pythagorean Fuzzy Set (PFS), Neutrosophic Fuzzy Set (NFS), Pythagorean Neutrosophic Fuzzy Set (PNFS), Pythagorean Octagonal Neutrosophic Fuzzy Number (PONFN), Interval Valued Pythagorean Octagonal Neutrosophic Fuzzy Number (IVPONFN).

Introduction

In 1975, L.A.Zadesh[58] was discovered the notion of Fuzzy Set theory. The degree of membership function was suggested and deeply discussed in fuzzy set (*FS*) theory. Fuzzy Set theory has been applied in various fields like Homoeopathic verdict, Computer Science, and Fuzzy Algebra. In 1986, Atanassov [10] developed the idea of Intuitionistic Fuzzy Set (*IFS*) in which degree of membership as well as the degree of non-membership function are discussed. Generalization of fuzzy set theory is called the Intuitionistic Fuzzy Set. Intuitionistic Fuzzy Set theory applied in Engineering, Management Science and Computer Science [1, 6, 7, 26, 27, 19, 61, 47, 48, 49, 50, 51, 52, 40].

Atanassov also presented mathematical operations such as algebraic product, sum, union, intersection and complement [12, 9]. Pseudo fixed topics of all operators defined over the Intuitionistic Fuzzy Set [11] is also developed by him. In 1986, many scholars [7] have completed works in the field of Intuitionistic Fuzzy Set and its presentations. Many scholars [25, 35, 53, 57] further extended the concept of Intuitionistic Fuzzy Sets to introduce Interval Valued Intuitionistic Fuzzy Sets (*IVIFSs*), which enhances greatly the representation ability of uncertainty than *IFs*. However, the domain of Intuitionistic Fuzzy Sets and Interval Valued Intuitionistic Fuzzy Sets which are used to indicate the certain criterion does or does not belong to some fuzzy concepts.

Yager [54-56] developed the Pythagorean Fuzzy Set (*PFS*), which gives the accurate solutions in ambiguous and imprecise environments. When compared to *FS* and *IFS*, the Pythagorean fuzzy number becomes a more comprehensive computational model. Late, Smarandache [45] introduced the Neutrosophic Set and Neutrosophic probability and their logic which consists of three logics such as truth, indeterminacy, and falsity-membership degree. Interval Valued Fuzzy Sets were implemented by Zadeh [59]. An interval-valued membership function defines an Interval Valued Fuzzy Set (*IVFS*). *IVFs* are a subset of L-fuzzy sets [23] and type-2 fuzzy sets [20].

The Interval Valued Pythagorean Fuzzy Set (*IVPFS*) is an extension of PFS [36]. Numerous real-world issues related to decision-making can be expressed using the interval number within [*0, 1*]. Set's membership as well as its absence degrees to have an interval value Interval Valued Pythagorean Fuzzy Sets (*IVPFS*). It should be noted that when the upper and lower limits of the interval values are identical, *IVPFS* becomes *PFS*, indicating that the latter is a special case of the former [60, 37].

The origin of transportation models dates back to 1941 when F.L. Hitchcock published a paper entitled 'The Distribution of a Product from Several Sources to Numerous Localities.' The above paper did more contribution in finding the solution to transportation problems. In 1947, T.C. Koopmans implemented the article called 'Optimum Utilization of the Transportation System'. Above two contributions are mainly responsible for the development of transportation models which play vital role in shipping industry. Because, each shipping source has a certain capacity and each destination has a certain requirement associated with a certain cost of shipping from the sources to the destinations [39].

Researchers from different fields developed triangular, trapezoidal and pentagonal neutrosophic numbers, and presented the notions, properties along with applications in different fields [15, 17, 18]. The de-neutrosophication technique of pentagonal number and its applications are presented by [14, 16, 13, 21].

Sreeja T.S and Jeyanthi .V presents an application of Octagonal Neutrosophic Number and discuss with a score function [46].

Anandhi .M, and D.Arthi, works with a single valued octagonal neutrosophic number and introduces a new approach for using a score function [8].

Jansi, R., Mohana, K., Smarandache. F, contribute his work with properties of Pythagorean Neutrosophic Set [24].

Singh A, Bhat S.A represents a score and accuracy function for Neutrosophic Sets [44].

Scientists from different areas investigated the various properties and fluctuations of neutrosophic numbers and the properties of correlation between these numbers [14, 18]. The applications in decision-making in different fields like phone selection [42-43], games prediction [39], supplier selection [2, 4-5], medical [3], personnel selection [28-29].

Octagonal neutrosophic number and its types are presented by [41] in his recent work. The graphical representation and properties are yet to be defined while dealing with the concept of octagonal neutrosophic number a decision-maker can solve more fluctuations because they have more edges as compare to pentagonal.

Narmada Devi and Sowmiya [32, 33] introduced the Octagonal neutrosophic fuzzy number in game and sequencing problem.

Narmada Devi et al. [34] proposed the suitable waste to energy technology for India using MULTIMOORA method. The majority of WTE options were identified under different MCDMs using various fuzzy sets.

Narmada Devi .R, works with Different types of graphs on neutrosophic concept [30, 31].

The article aims to introduce an Interval Valued Pythagorean Octagonal Neutrosophic Fuzzy Number along with operational rules and score function for IVPONFN. After that, formulate the IVPONFN transportation problem will take place. Suitable examples have given wherever it required.

2. Preliminaries

Definition 2.1. [54-56]

Consider Z to be a universal set. Then a Pythagorean Fuzzy Set *P*, which is a set of ordered pairs over Z, is defined as follows:

$$P = \{ (z, \alpha(z), \beta(z)) |\ z \in Z\}$$

where $\alpha_P(z) : Z \to [0,1]$ and $\beta_P(z) : Z \to [0,1]$ define the degree of membership and non-membership, respectively, of the element $z \in Z$ to *P*, which is a subset of *z*, and for every $z \in Z$:

$$0 \leq \left(\alpha_P(z)\right)^2 + \left(\beta_P(z)\right)^2 \leq 1$$

Suppose $\left(\alpha_P(z)\right)^2 + \left(\beta_P(z)\right)^2 \leq 1$ then there is a degree of indeterminacy of $z \in Z$ to *P* defined by $\alpha_P(z)$: $\sqrt{1 - [\left(\alpha_P(z)\right)^2 + \left(\beta_P(z)\right)^2]}$ and $\alpha_P(z) \in [0, 1]$. In what follows, $\left(\alpha_P(z)\right)^2 + \left(\beta_P(z)\right)^2 = 1$. Otherwise, $\alpha_P(z) = 0$ whenever $\left(\alpha_P(z)\right)^2 + \left(\beta_P(z)\right)^2 = 1$.

Definition 2.2. [45]

Let Z be a non-empty set. A Neutrosophic Fuzzy Set N on Z is an object of the form:

$$N = \{ (z, \alpha_N(z), \gamma_N(z), \beta_N(z)) : z \in Z\}$$

Where $\alpha_N(z), \gamma_N(z), \beta_N(z) \in [0,1]$, $0 \leq \alpha_N(z) + \gamma_N(z) + \beta_N(z) \leq 3$ for all $z \in Z$, $\alpha_N(z)$, $\gamma_N(z)$, $\beta_N(z)$ are degrees of membership, indeterminacy and non-membership, respectively. Here, $\alpha_N(z)$ and $\beta_N(z)$ are dependent component and $\gamma_N(z)$ is an independent components.

Definition 2.3. [36]

Let Z be a universal set. A Pythagorean Neutrosophic fuzzy set (*PNFS*) with *T* and *F* are dependent Neutrosophic components *D* on *Z* is an object of the form

$$D = \{ (z, \alpha_D(z), \gamma_D(z), \beta_D(z)) : z \in Z\}$$

where $\alpha_D(z), \gamma_D(z), \beta_D(z) \in [0,1]$, $0 \leq (\alpha_D(z))^2 + (\gamma_D(z))^2 + (\beta_D(z))^2 \leq 2$, for all $z \in Z$, $\alpha_D(z), \gamma_D(z)$ and $\beta_D(z)$ are degrees of membership, indeterminacy, non-membership, respectively. Here, $\alpha_D(z)$ and $\beta_D(z)$ are dependent component and $\gamma_D(z)$ is an independent components.

Definition 2.4. [37, 22]

Let *Z* be a universal set. An Interval-Valued Pythagorean Neutrosophic Fuzzy Set (PNFS) with T and F are dependent Neutrosophic components *C* on *Z* is an object of the form

$$C = \{(z, [\alpha_C^L(z), \alpha_C^U(z)] , [\gamma_C^L(z), \gamma_C^U(z)] , [\beta_C^L(z), \beta_C^U(z)] : z \in Z\}$$

Where $[\alpha_C^L(z), \alpha_C^U(z)] , [\gamma_C^L(z), \gamma_C^U(z)] , [\beta_C^L(z), \beta_C^U(z)] \in [0, 1]$,

$$0 \leq [\frac{\alpha_C^L(z) + \alpha_C^U(z)}{2}]^2 , [\frac{\gamma_C^L(z) + \gamma_C^U(z)}{2}]^2 , [\frac{\beta_C^L(z) + \beta_C^U(z)}{2}]^2 \leq 2$$

for all $z \in Z$, $[\alpha_C^L(z), \alpha_C^U(z)]$ is the degree of membership. $[\gamma_C^L(z), \gamma_C^U(z)]$ is the degree of indeterminacy and $[\beta_C^L(z), \beta_C^U(z)]$ is the degree of non-membership.

Linguistic term	Membership values	Indeterminacy values	Non – membership values
Extremely Low	[0.1, 0.2]	[0.5, 0.6]	[0.8, 0.9]
Medium	[0.2, 0.4]	[0.4, 0.5]	[0.6, 0.8]
Medium High	[0.4, 0.6]	[0.3, 0.4]	[0.4, 0.6]
Very High	[0.6, 0.8]	[0.2, 0.3]	[0.2, 0.4]
Extremely High	[0.8, 0.9]	[0.1, 0.2]	[0.1, 0.2]

Table 1. Interval-Valued Pythagorean Neutrosophic Fuzzy Linguistic Table

3. Operations on Interval Valued Pythagorean Octagonal Neutrosophic Fuzzy Number (IVPONFN)

Definition 3.1

Let $p = ([p_a, p_b, p_c, p_d, p_e, p_f, p_g, p_h]; \alpha, \beta, \gamma) = ([p_a, p_b, p_c, p_d, p_e, p_f, p_g, p_h]; [\alpha_1, \alpha_2], [\beta_1, \beta_2], [\gamma_1, \gamma_2])$ be an Interval Valued Pythagorean Octagonal Neutrosophic Fuzzy Number where $\alpha = [\alpha_1, \alpha_2]$ and $\beta = [\beta_1, \beta_2]$ and $\gamma = [\gamma_1, \gamma_2]$ represent an interval valued, hence $\alpha \subset [0,1]$ And $\beta \subset [0,1]$ and $\gamma \subset [0,1]$ such that $0 \leq \left(\frac{\alpha_1+\alpha_2}{2}\right)^2 + \left(\frac{\beta_1+\beta_2}{2}\right)^2 + \left(\frac{\gamma_1+\gamma_2}{2}\right)^2 \leq 2$.

Definition 3.2

Let $p_1 = ([p'_a, p'_b, p'_c, p'_d, p'_e, p'_f, p'_g, p'_h]; [\alpha'_1, \alpha'_2], [\beta'_1, \beta'_2], [\gamma'_1, \gamma'_2])$ and $p_2 = ([p''_a, p''_b, p''_c, p''_d, p''_e, p''_f, p''_g, p''_h]; [\alpha''_1, \alpha''_2], [\beta''_1, \beta''_2], [\gamma''_1, \gamma''_2])$ be any two IVPONFN and $\delta > 0$.

Then

1. $$p_1 \oplus p_2 = \begin{cases} [p'_a + p''_a, p'_b + p''_b, p'_c + p''_c, p'_d + p''_d, p'_e + p''_e, p'_f + p''_f, p'_g + p''_g, p'_h + p''_h]; \\ [\sqrt{(\alpha'_1)^2 + (\alpha''_1)^2 - (\alpha'_1\alpha''_1)^2} \times \sqrt{(\alpha'_2)^2 + (\alpha''_2)^2 - (\alpha'_2\alpha''_2)^2}, \\ [(\beta'_1\beta''_1, \beta'_2\beta''_2)][(\gamma'_1\gamma''_1, \gamma'_2\gamma''_2)] \end{cases}$$

2. $$p_1 \otimes p_2 = \begin{cases} [p'_a \times p''_a, p'_b \times p''_b, p'_c \times p''_c, p'_d \times p''_d, p'_e \times p''_e, p'_f \times p''_f, p'_g \times p''_g, p'_h \times p''_h]; \\ [\alpha'_1 \times \alpha''_1, \alpha'_2 \times \alpha''_2][\sqrt{(\beta'_1)^2 + (\beta''_1)^2 - (\beta'_1 \times \beta''_1)^2}, \\ \sqrt{(\beta'_2)^2 + (\beta''_2)^2 - (\beta'_2 \times \beta''_2)^2}], [\sqrt{(\gamma'_1)^2 + (\gamma''_1)^2 - (\gamma'_1 \times \gamma''_1)^2}, \\ \sqrt{(\gamma'_2)^2 + (\gamma''_2)^2 - (\gamma'_2 \times \gamma''_2)^2}] \end{cases}$$

3. $$\delta p_1 = \begin{cases} [\delta p'_a, \delta p'_b, \delta p'_c, \delta p'_d, \delta p'_e, \delta p'_f, \delta p'_g, \delta p'_h]; [\sqrt{1 - (1 - (\alpha'_1)^2)^\delta}, \sqrt{1 - (1 - (\alpha'_2)^2)^\delta}], \\ [((\beta'_1)^\delta, (\beta'_2)^\delta)][((\gamma'_1)^\delta, (\gamma'_2)^\delta)] \end{cases}$$

4. $$p_1^\delta = \begin{cases} [(p'_a)^\delta, (p'_b)^\delta, (p'_c)^\delta, (p'_d)^\delta, (p'_e)^\delta, (p'_f)^\delta, (p'_g)^\delta, (p'_h)^\delta]; [(\alpha'_1)^\delta, (\alpha'_2)^\delta], \\ [\sqrt{1 - (1 - (\beta'_1)^2)^\delta}, \sqrt{1 - (1 - (\beta'_2)^2)^\delta}], \\ [\sqrt{1 - (1 - (\gamma'_1)^2)^\delta}, \sqrt{1 - (1 - (\gamma'_2)^2)^\delta}] \end{cases}$$

Definition 3.3

Let $p = ([p_a, p_b, p_c, p_d, p_e, p_f, p_g, p_h]; [\alpha_1, \alpha_2], [\beta_1, \beta_2], [\gamma_1, \gamma_2])$ be an Interval Valued Pythagorean Octagonal Neutrosophic Fuzzy Number. Then a Score function S can be defined as follows:

$$S(p) = \frac{1}{32}(p_a + p_b + p_c + p_d + p_e + p_f + p_g + p_h)[2 + (\alpha_1 + \alpha_2) - 2(\beta_1 + \beta_2) - (\gamma_1 + \gamma_2)]$$

4. Mathematical Formulation of Interval Valued Pythagorean Octagonal Neutrosophic Fuzzy Number (IVPONFN) Transportation problem

The transportation problem has been given in the form of $m \times n$ IVPONFN cost table $[C_{ij}]$ is tabulated below (Table 2):

	D_1	D_2	⋮	D_j	⋮	D_n	**Quantities**
O_1	C_{11}	C_{12}		C_{1j}		C_{1n}	$\tilde{a}_1$
⋮	⋮	⋮		⋮		⋮	⋮
O_i	C_{i1}	C_{i2}		C_{ij}		C_{in}	$\tilde{a}_i$
⋮	⋮	⋮		⋮		⋮	⋮
O_m	C_{m1}	C_{m2}		C_{mj}		C_{mn}	$\tilde{a}_m$
Requirements	$\tilde{b}_1$	$\tilde{b}_2$		$\tilde{b}_j$		$\tilde{b}_n$	

Table 2: Transportation Cost Matrix

The cost C_{ij} are taken as IVPONFN S.

$$\tilde{C}_{ij}^{IPN} = ((\tilde{C}_{ij}^{IPN}, \tilde{C}_{ij}^{IPN}, \tilde{C}_{ij}^{IPN}, \tilde{C}_{ij}^{IPN})[\mu_{ij}^L, \mu_{ij}^U], [v_{ij}^L, v_{ij}^U], [w_{ij}^L, w_{ij}^U])$$

The target is to minimize the IVPONFN cost induced transportation successfully. Let us presume that there are *m* quantities at the sources and *n* requirements at the destination. Let $\tilde{a}_i^{IPN}$ the IVPONFN quantities at *i* and $\tilde{b}_j^{IPN}$ IVPONFN requirement at *j* be the unit cost IVPONFN transportation cost from source *i* to destination *j* and $\tilde{X}_{ij}$ be the number of units shifted from source *i* to destination *j*. The IVPONFN transportation problem can be mathematically expressed as

$$min\ \tilde{z} = \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{C}_{ij}^{IPN} \tilde{X}_{ij}$$

subject to the constraints, $\sum_{j=1}^{n} \tilde{X}_{ij} = \tilde{a}_i$, $i = 1$ to *m* and $\sum_{i=1}^{m} \tilde{X}_{ij} = \tilde{b}_j$, $j = 1$ to *n*

The IVPONFN problem is said to be balanced

$$\sum_{j=1}^{n} \tilde{a}_i = \sum_{j=1}^{n} \tilde{b}_j$$

(i.e., if the total supply from all the sources is equal to the demand of the requirement)

4.1. A Mathematical approach for IVPONFN:

This section provides step by step procedure for a Mathematical approach to the IVPONFN transportation problem which leads as:

Step 1: Using the ranking function described, obtain the rank for each cell of the chosen generalized IVPONFN cost matrix.

Step 2: Test the generalized IVPONFN transportation problem is balanced, if it is balanced, Proceed to step 4, otherwise move on to step 3.

Step 3: Use zero generalized single-valued dummy rows of dummy columns. To construct a balanced one, Natural number must be added.

Step 4: Find the initial basic feasible solution by following the NWC, LCM, VAM approach and if it is degeneracy check the optimality test by following a modified distribution approach to obtain the optimal solution.

Step 5: Using generalized IVPONFN cost by adding the optimal generalized IVPONFN cost which minimizes the Transportation cost.

5. Application of Interval Valued Pythagorean Octagonal Neutrosophic Fuzzy Number (IVPONFN)

The above Mathematical approach is illustrated in the following transportation problem, based on IVPONFN. To obtain the best optimal solution. Here the problem is considered as Chemical Factory requirement to get the best optimal solution as well as the optimal decision to the stack holders. A Factory has three branches A_1, A_2, A_3 from which it supplies to three destinations D_1, D_2, D_3 .Among the three branches of 8 members in each branches whose transportation cost between the various destination is considered interms of Linguistic variable like Extremely Low, Medium, Medium High, Very High, Extremely High according to the transportation cost of this branches to the destination and so on. Based on the Linguistic variable of the group membership indeterminacy, non-membership are defined and is given by the following table .Calculate the optimal cost depending on the appropriate. Date giving the chemicals to distribute the destinations shifting cost, quantities available at each branches and quantities required at each destination. Due to uncertain situations, all the costs of the problem as IVPONFN.

Using the proposed ranking function the transportation cost from branches to the destinations has defuzzified as a crisp number and applied the Mathematical approach to the problem to get the best optimal solution.

Table 3

	D_1	D_2	D_3	Supply
A_1	(2.1, 2.2, 2.3, 2.4, 2.5, 2.7, 2.8, 2.9) [0.2, 0.4] [0.3, 0.4] [0.4, 0.6]	(2.1, 2.2, 2.4, 2.7, 2.9, 3.1, 3.3, 3.5) [0.2, 0.4] [0.3, 0.4] [0.4, 0.6]	(4.1, 4.2, 4.3, 4.4, 4.6, 4.7, 4.8, 5.0) [0.6, 0.8] [0.2, 0.3] [0.2, 0.4]	(1.0, 1.2, 1.3, 1.5, 1.6, 1.8, 2.0, 2.2) [0.2, 0.4] [0.3, 0.4] [0.4, 0.6]
A_2	(5.1, 5.2, 5.5, 5.6, 5.7, 5.8, 5.9, 6.0) [0.8, 0.9] [0.2, 0.3] [0.1, 0.2]	(3.1, 3.2, 3.4, 3.5, 3.7, 3.8, 3.9, 4.1) [0.4, 0.6] [0.3, 0.4] [0.4, 0.6]	(4.1, 4.2, 4.3, 4.6, 4.7, 4.9, 5.2, 5.4) [0.6, 0.8] [0.2, 0.3] [0.2, 0.4]	(1.5, 1.7, 1.9, 2.2, 2.5, 2.7, 2.9, 3.2) [0.4, 0.6] [0.3, 0.4] [0.4, 0.6]
A_3	(5.3, 5.5, 5.7, 5.9, 6.2, 6.4, 6.5, 6.8) [0.8, 0.9] [0.1, 0.2] [0.1, 0.2]	(6.1, 6.2, 6.3, 6.5, 6.7, 6.9, 7.0, 7.2) [0.8, 0.9] [0.1, 0.2] [0.1, 0.2]	(3.1, 3.3, 3.4, 3.6, 3.7, 3.8, 4.0, 4.2) [0.4, 0.6] [0.3, 0.4] [0.4, 0.6]	(3.0, 3.3, 3.6, 3.8, 4.0, 4.2, 4.5, 5.0) [0.8, 0.9] [0.1, 0.2] [0.1, 0.2]
Demand	(2.0, 2.2, 2.4, 2.5, 2.7, 2.8, 3.2, 3.4) [0.4, 0.6] [0.3, 0.4] [0.4, 0.6]	(2.2, 2.4, 2.6, 2.8, 3.0, 3.3, 3.5, 3.8) [0.6, 0.8] [0.2, 0.3] [0.2, 0.4]	(2.0, 2.5, 3.2, 3.4, 3.5, 3.9, 4.0, 4.5) [0.6, 0.8] [0.2, 0.3] [0.2, 0.4]	

Solution:

To convert IVPONFN to crisp number by using a proposed score function S(p) for the table 3 and it is given as score matrix as below.

	D_1	D_2	D_3	a_i
A_1	0.12	0.13	2.03	0.79
A_2	3.36	0.54	2.10	3.49
A_3	4.23	4.63	0.55	27.48
b_j	3.98	13.28	15.19	

We have to check Supply = Demand

0.79 + 3.49 + 27.48 = 3.98 + 13.28 + 15.19

$31.76 \neq 32.45$

It is unbalanced transportation problem

Now we add Dummy Row

			a_i
0.12	0.13	2.03	0.79
3.36	0.54	2.10	3.49
4.23	4.63	0.55	27.48

0	0	0	0.69
b_j 3.98	13.28	15.19	

(i) North – West Corner Method

Step 1:

Consider a North corner is C_{11}

min (a_1, b_1)= 0.79 occurred in a_1

Delete first row.

Step 2:

Consider a North corner is C_{21}

min (a_2, b_1)= 3.19 occurred in b_1

Delete first column.

Step 3:

Consider a North corner is C_{22}

min (a_2, b_2)= 0.30 occurred in a_1

Delete second row.

Step 4:

Consider a North corner is C_{32}

min (a_3, b_2)= 12.98 occurred in b_2

Delete second column.

Step 5:

Consider a North corner is C_{33}

min (a_3, b_3)= 4.50 occurred in a_3

Delete third row.

Step 6:

Consider the remaining cost is C_{43} (i.e., 0)

The Optimal allocation for this problem is

0.12 (0.79)	0.13	2.03	0.79
3.36 (3.19)	0.54 (0.30)	2.10	3.49
4.23	4.63 (12.98)	0.55 (14.50)	27.48
3.98	13.28	15.19	

The Optimal Transportation cost is

= 0.79(0.12) + 3.36(3.19) + 0.54(0.30) + 4.63(12.98) + 0.55(14.50)

= 0.10 +10.72 + 0.16+ 60.10 + 7.98

= 79.06

The Optimal Transportation cost is 79.06

(ii) Least Cost Method

				a_i
	0.12	0.13	2.03	0.79
	3.36	0.54	2.10	3.49
	4.23	4.63	0.55	27.48
	0	0	0	0.69
b_j	3.98	13.28	15.19	

Step 1:

min $(C_{ij}) = 0$ occurred in C_{41}

min (a_4, b_1)= 0.69 occurred in a_4

Delete fourth row.

Step 2:

min (C_{ij}) =0.12 occurred in C_{11}

min (a_1, b_1)= 0.79 occurred in a_1

Delete first row.

Step 3:

min (C_{ij}) =0.54 occurred in C_{22}

min (a_2, b_2)= (3.49, 13.28) = 3.49 occurred in a_2

Delete second row.

Step 4:

min (C_{ij}) =0.55 occurred in C_{33}

min (a_3, b_3)= 15.19 occurred in b_3

Delete third column.

Step 5:

min (C_{ij}) =4.23 occurred in C_{31}

min (a_3, b_3)= 2.50 occurred in b_1

Delete first column.

Step 6:

The remaining cost is 4.63 occurred in C_{32}.

The Optimal allocation for this problem is

0.12 (0.79)	0.13	2.03	0.79

3.36	3.49 0.54	2.10	3.49
2.50 4.23	9.79 4.63	15.19 0.55	27.48
3.98	13.28	15.19	

The Optimal Transportation cost is

= 0.12(0.79) + 0.54(3.49) + 4.23(2.50) + 4.63(9.79) + 0.55(15.19)

= 0.10 +1.89 + 10.58 + 45.33 + 8.36

= 66.26

The Optimal Transportation cost is 66.26

(iii) Vogel's Approximation Method (VAM)

				a_i
	0.12	0.13	2.03	0.79
	3.36	0.54	2.10	3.49
	4.23	4.63	0.55	27.48
	0	0	0	0.69
b_j	3.98	13.28	15.19	

Step 1:

Find the Penality cost namely the difference between the lowest and second lowest cost in each row and each column and choose the maximum penality

				a_i					
	0.12	0.79 0.13	2.03	0.79	0.01	0.01	0.01	0.13	0.13
	3.36	3.49 0.54	2.10	3.49	1.56	2.82	-	-	-
	3.98 4.23	8.31 4.63	15.19 0.55	27.48	3.68	0.4	0.40	4.63	-
	0	0.69 0	0	0.69	0	0	0	0	0
b_j	3.98	13.28	15.19						
	0.12	0.13	0.55						

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0.12	0.13	-
0.12	0.13	-
-	0.13	-
-	0.13	-

Step 2:

The Maximum penality is 3.68 which is occurred in a_3. min (C_{31}, C_{32}, C_{33}) = 0.55
Which is occurred in C_{33}.
min (a_3, b_3) = 15.19 occurred in b_3
Delete third column.

Step 3:

Repeat the procedure the Maximum penality is 2.82 occurred in a_2.
min (C_{21}, C_{22}) = 0.54 occurred in C_{22}.
min (a_2, b_2) = 3.49 occurred in a_2
Delete second row.

Step 4:

Repeat the procedure the Maximum penality is 0.40 occurred in a_3.
min (C_{31}, C_{32}) = 4.23 occurred in C_{31}.
min (a_3, b_1) = 3.98 occurred in b_1
Delete first column.

Step 5:

Repeat the procedure the Maximum penality is 4.63 occurred in a_3.
4.63 occurred in C_{32}.
min (a_3, b_2) = 8.31 occurred in a_3
Delete third row.

Step 6:

Repeat the procedure the Maximum penality is 0.13 occurred in a_1.
0.13is occurred in C_{12}.
min (a_1, b_2) = 0.79 occurred in a_1
Delete first row.

Step 7:

The remaining allocation is 0.69 for C_{42}.

The optimal allocation is

				a_i
	0.12	0.79 0.13	2.03	0.79
	3.36	3.49 0.54	2.10	3.49
	3.98 4.23	8.31 4.63	15.19 0.55	27.48
b_j	3.98	13.28	15.19	

The Optimal Transportation cost is

= 0.13(0.79) + 0.54(3.49) + 4.23(3.98) + 4.63(8.31) + 0.55(15.19)

= 0.10 +1.89 + 16.84 + 38.48 + 8.36

= 65.67

The Transportation cost is

NWC – 79.06 Rs

LCM – 66.26 Rs

VAM – 65.67 Rs

Conclusion

For addressing the situation where inconsistencies and insufficient data exist, Neutrosophic Set will be useful. This article suggests a Mathematical Neutrosophic Approach using ranking function for Transportation Problem. Since this method gives the optimal and accurate solutions, stack holders always use interval valued neutrosophic approach for taking precious decision.

References:

[1] Abbas S. E, M. A. Hebeshi and I. M. Taha, On upper and lower contra-continuous fuzzy multifunctions, Punjab Univ. J. Math. 47, (2017) 105-117.

[2] Abdel-Baset, M., Chang, V., Gamal, A., & Smarandache, F. An integrated neutrosophic ANP and VIKOR method for achieving sustainable supplier selection: A case study in the importing field. Computers in Industry, vol 106, pp. 94-110, 2019.

[3] Abdel-Basset, M., Manogaran, G., Gamal, A., & Smarandache, F. A group decision-making framework based on the neutrosophic TOPSIS approach for smart medical device selection. Journal of medical systems, vol 43, issue 2, pp. 38-43, 2019.

[4] Abdel-Basset, M., Manogaran, G., Gamal, A., & Smarandache, F. A hybrid approach of neutrosophic sets and DEMATEL method for developing supplier selection criteria. Design Automation for Embedded Systems, pp.1-22, 2018.

[5] Abdel-Basset, M., Saleh, M., Gamal, A., & Smarandache, F. An approach of the TOPSIS technique for developing supplier selection with group decision making under type-2 neutrosophic number. Applied Soft Computing, vol77, pp. 438-452, 2019.

[6] Akram .M and G. Shahzadi, Certain caracterization of m-polar fuzzy graphs by level graphs, Punjab Univ. J. Math. 49, (2017) 1-12.

[7] Alamgir Khan .M and Sumitra, Common fixed point Theorems for converse commuting and OWC maps in fuzzy metric spaces, Punjab Univ. J. Math. 44, (2016) 57-63.

[8] Anandhi .M, and D.Arthi, "Octagonal Neutrosophic Numbers and Their Application in Transportation Problem Environment using Russell's Approximation Method", International Journal of Mechanical Engineering, ISSN: 0974-5823, vol. 7 No. 4 April 2022.

[9] Atanassov .K, An equality between intuitionistic fuzzy sets, Fuzzy Sets Syst. 79, (1996) 257-258.

[10] Atanassov .K, Intuitionstic fuzzy sets, Fuzzy Sets Syst. 20, (1986) 87-96.

[11] Atanassov .K, Remarks on the intuitionistic fuzzy sets-III, Fuzzy Sets Syst. (1995) 401-402.

[12] Atanassov .K, G. Pasi, R. R. Yager, Intuitionistic fuzzy interpretations of multi-criteria multi-person and multi-measurement tool decision making, Int. J. Syst. Sci. 36, (2005) 859-868.

[13] Chakraborty, A. "A New Score Function of Pentagonal Neutrosophic Number and its Application in Networking Problem," International Journal of Neutrosophic Science, Volume 1, Issue 1, pp. 40-51, 2020.

[14] Chakraborty. A., Broumi. S, P.K Singh,"Some properties of Pentagonal Neutrosophic Numbers and its Applications in Transportation Problem Environment," Neutrosophic Sets and Systems, vol.28, pp.200-215, 2019.

[15] Chakraborty, A.; Mondal, S. P.; Ahmadian, A.; Senu, N.; Alam, S.; and Salahshour, S.;

Different Forms of Triangular Neutrosophic Numbers, De-Neutrosophication Techniques, and their Applications, Symmetry, vol 10, 327, 2018.; doi:10.3390/sym10080327

[16] Chakraborty. A., S. Mondal, S. Broumi, "De-Neutrosophication technique of pentagonal neutrosophic number and application in minimal spanning tree," Neutrosophic Sets and Systems, vol. 29, pp. 1-18,2019. doi: 10.5281/zenodo.3514383.

[17] Chakraborty, A.; Mondal, S. P.;Mahata, A.; Alam, S.; Different linear and non-linear form of Trapezoidal Neutrosophic Numbers, De-Neutrosophication Techniques and its Application in time cost optimization technique, sequencing problem; RAIRO Operation Research, vol 11, 327, 2018. doi:10.1051/ro/2019090.

[18] Chakraborty. A., Sankar P. Mondal, Shariful A. Ali A., Norazak S., Debashis De. and Soheil S., The Pentagonal Fuzzy Number: Its Different Representations, Properties, Ranking, Defuzzification and Application in Game Problems, Symmetry, vol 10, pp 248, 2018.

[19] Cornelis .C, G. Deschrijver and E. E. Kerre, Implication in intuitionistic fuzzy and interval-valued fuzzy set theory construction, classification, application, Int. J. Approx. Reason. 35, (2004) 55-95

[20] Dubois, D., & Prade, H. (2005, September). Interval-valued Fuzzy Sets, Possibility Theory and Imprecise Probability. In EUSFLAT Conf. (pp. 314-319).

[21] Edalatpanah, S. A., "A Direct Model for Triangular Neutrosophic Linear Programming," International Journal of Neutrosophic Science, Volume 1, Issue 1, pp. 19-28, 2020.

[22] Garg, H. (2017). A novel improved accuracy function for interval valued Pythagorean fuzzy sets and its applications in the decision-making process. International Journal of Intelligent Systems, 32(12), 1247-1260.

[23] Goguen, J. A. (1967). L-fuzzy sets. Journal of mathematical analysis and applications, 18(1), 145-174.

[24] Jansi, R., Mohana, K., Smarandache, F. (2019). Correlation measure for pythagorean neutrosophic sets with t and f as dependent neutrosophic components. Infinite Study.

[25] Jun .Ye, Multicriteria fuzzy decision-making method using entropy weights-based correlation coefficients of interval-valued intuitionistic fuzzy sets, Applied Mathematical Modelling. 34, (2010) 3864-3870.

[26] Li D. F, Closeness coefficient based nonlinear programming method form interval-valued intuitionistic fuzzy multiattribute decision making with incomplete preference information, Appl. Soft Comput. 11, (2011) 3042-3418.

[27] Merigó J. M, A. M. Gil-Lafuente, Fuzzy induced generalized aggregation operators and its application in multi-person decision making, Expert Syst. Appl. 38, (2011) 9761-9772.

[28] Mondal .K, and S. Pramanik. Neutrosophic decision making model of school choice. Neutrosophic Sets and Systems, vol 7, pp. 62-68, 2015.

[29] Nabeeh, N. A., Smarandache, F., Abdel-Basset, M., El-Ghareeb, H. A., & Aboelfetouh, A. An Integrated Neutrosophic-TOPSIS Approach and Its Application to Personnel Selection: A New Trend in Brain Processing and Analysis. IEEE Access, vol 7, pp. 29734-29744. 2019.

[30] Narmada Devi .R, 'Minimal domination via Neutrosophic over Graphs', AIP Conference Proceedings, 2277, 100019: https://doi.org/10.106375.0023568.

[31] Narmada Devi .R and Davaseelan R 'New type of Neutrosophic Off Graphs'. Advanced in Mathematics Scientific Journal, Vol. 9, No. 3, pp.1331-1338.

[32] Narmada Devi .R., & Sowmiya, S. (2022). On solving game problem using octagonal neutrosophic fuzzy number, International Journal of Operational Research. (Accepted)

[33] Narmada Devi .R., & Sowmiya, S , Solution of sequencing problem by using neutrosophic octagonal number, AIP Conference Proceedings 2516, 340004(2022); Published Online: 30 November 2022 https://doi.org/10.1063/5.0108810

[34] Narmada Devi .R, Sowmiya, S, Anuja .A,Selecting the Suitable Waste to Energy Technology for India Using MULTIMOORA Method under Pythagorean Neutrosophic

Fuzzy Logic, Neutrosophic Sets and Systems, Vol. 56, 2023, pp. 276-290. DOI: 10.5281/zenodo.8194803
[35] Park J. H, Y. Park, Y. C. Kwun and X. G. Tan, Extension of the topsis method for decision making problems under interval-valued intuitionistic fuzzy environment, Appl. Math. Model. 35, (2011) 2544-2556.
[36] Peng, X. (2019). New operations for interval-valued Pythagorean fuzzy set. Scientia iranica, 26(2), 1049- 1076.
[37] Peng, X., & Yang, Y. (2016). Fundamental properties of interval-valued Pythagorean fuzzy aggregation operators. International Journal of Intelligent Systems. 31(5), 444-487.
[38] Prem Kumar Gupta .Er, Dr.D.S. Hira , Operations Research.
[39] Riaz. M., Saqlain. M. and Saeed. M. Application of Generalized Fuzzy TOPSIS in Decision Making for Neutrosophic Soft set to Predict the Champion of FIFA 2018: A Mathematical Analysis, Punjab University Journal of Mathematics, vol 51(8), pp.111-126, 2019.
[40] Sajjad Ali Khan .M, K. Rahman, A. Fahm and M. Shakeel, Generalized (e, e V qk)-fuzzy quasi-ideals in semigroups, Punjab Univ. J. Math. 50, (2018) 35-53.
[41] Saqlain .M, A. Hamza, and S. Farooq, "Linear and Non-Linear Octagonal Neutrosophic Numbers: Its Representation, α-Cut and Applications," International Journal of Neutrosophic Science, vol. 3, no. 1, pp.29–43, 2020.
[42] Saqlain, M., Jafar, M. N., Riaz, M. A New Approach of Neutrosophic Soft Set with Generalized Fuzzy TOPSIS in Application of Smart Phone Selection, Neutrosophic Sets and Systems, vol. 32, pp. 307-316, 2020. DOI: 10.5281/zenodo.3723161.
[43] Saqlain. M., Jafar. N. and Riffat. A., Smart phone selection by consumers' in Pakistan: FMCGDM fuzzy multiple criteria group decision making approach, Gomal University Journal of Research, vol 34(1), pp. 27-31, 2018.
[44] Singh, A., Bhat, S. A. (2021). A novel score and accuracy function for neutrosophic sets and their real-world applications to multi-criteria decision-making process. Neutrosophic Sets and Systems, 41, 168-197.
[45] Smarandache, F. (1998). Neutrosophy/neutrosophic probability, set and logic. American Research Press,Rehoboth.
[46] Sreeja.TS, Jeyanthi.V, A New Algorithm for Solving Shortest Path Problems using Octagonal Neutrosophic Numbers, Test Engineering and Management, Nov-Dec-2019, Pg no: 4679-4685.
[47] Wei G. W, Some induced geometric aggregation operators with intuitionistic fuzzy information and their application to group decision making, Appl. Soft Comput. 10, (2010) 423-431.
[48] Wei G. W, Some arithmetic aggregation operators with intuitionistic trapezoidal fuzzy numbers and their application to group decision making, J. Comput. 5, (2010) 345–351.
[49] Xu Z. S, An overview of methods for determining owa weights, Int. J. Intell. Syst. 20, (2005) 843-865.
[50] Xu Z. S, and R. R. Yager, Some geometric aggregation operators based on intuitionistic fuzzy sets, Interna- tional Journal General System. 35, (2006) 417-433.
[51] Xu Z. S, Intuitionistic preference relations and their application in group decision making, Information Sciences. 177, (2007) 2363-2379.
[52] Xu Z. S, Intuitionistic fuzzy aggregation operators, IEEE Trans. Fuzzy Syst. 15, (2007) 1179-1187.
[53] Xu Z. S, A method based on distance measure for interval-valued intuitionistic fuzzy group decision making. Inform. Sci. 180, (2010) 181-190.
[54] Yager, R.R. (2013). Pythagorean fuzzy subsets. In Proceedings of the Joint IFSAWorld Congress and NAFIPS Annual Meeting, Edmonton, AB, Canada, June 24-28, 257-61.
[55] Yager, R. R. (2013). Pythagorean membership grades in multi-criteria decision making. IEEE Transactions on Fuzzy Systems, 22(4), 958-965.
[56] Yager, R. R., Abbasov, A. M. (2013). Pythagorean membership grades, complex numbers, and decision making. International Journal of Intelligent Systems, 28(5), 436-452.

[57] Ye .J, Multicriteria fuzzy decision-making method using entropy weights-based correlation coefficients of interval-valued intuitionistic fuzzy sets, Appl. Math. Model. 34, (2010) 3864-3870.
[58] Zadeh L. A, Fuzzy sets, Information. Control. 8, (1965) 338-353.
[59] Zadeh, L. A. (1975). The concept of a linguistic variable and its application to approximate reasoning-I. Information sciences, 8(3), 199-249.
[60] Zhang, X. (2016). Multicriteria Pythagorean fuzzy decision analysis: A hierarchical QUALIFLEX approach with the closeness index-based ranking methods. Information Sciences, 330, 104-124.
[61] Zhang H. Y, W. X. Zhang and W. Z. Wu, On characterization of generalized interval-valued fuzzy rough sets on two universes of discourse, Int. J. Approx. Reason. 51, (2009) 56-70.